Odds, odds ratios and related quantities
Numerical example, based on VitC study
Note: Reported values are rounded but prior calculations are not

Placebo group: 335 with cold, 411 total
$p_1 = 335 / 411 = 0.815$
Odds of a cold in placebo group $= 0.815/(1 - 0.815) = 335/76 = o_1 = 4.41$
Odds of not cold in placebo group $= (1 - 0.815)/0.815 = 76/335 = 1/o_1 = 0.227$

Vit C group: 302 with cold, 407 total
$p_2 = 302 / 407 = 0.742$
Odds of a cold in Vit C group $= 0.742/(1 - 0.742) = 302/105 = o_2 = 2.88$
Odds of not cold in Vit C group $= (1 - 0.42)/0.742 = 105/302 = 1/o_2 = 0.348$

Odds ratio and log odds ratio:
Odds ratio (as odds of cold in placebo / odds in Vit C): $o_1/o_2 = 4.41 / 2.88 = 1.53$
log odds ratio $= \log 1.53 = 0.427$
Odds ratio (as odds of cold in Vit C / odds in placebo): $o_2/o_1 = 2.88 / 4.41 = 0.652$
log odds ratio $= \log 0.652 = -0.427$
Odds ratio (as odds of not cold in placebo / odds in Vit C):
$(1/o_1)/(1/o_2) = 0.227 / 0.348 = 0.652$
log odds ratio $= \log 0.652 = -0.427$
Odds ratio (as odds of not cold in Vit C / odds in placebo):
$(1/o_2)/(1/o_1) = 0.348 / 0.227 = 1.53$
log odds ratio $= \log 1.53 = 0.427$

Checking the interpretation:
compare what your statement says to what the two probabilities say, e.g.
The odds of a cold in the Vit C group is 0.652 times that odds in the placebo group
OR: Vit C reduces the odds of a cold by 35% compared to the placebo group
Both say that the odds of a cold is lower in the Vit C group
Which means the probability of a cold must be lower in the Vit C Group

Check: P[cold | Vit C] = 0.742, P[cold | Placebo] = 0.815.
Yes, so statement about odds is "the right way around"

Inference on odds ratios: (expanded copy of last bit of week 6 notes)
log odds ratios are approximately normally distributed
Approximate se calculated from the 4 observed counts, O_{ij}:

$$\begin{aligned} se &\approx \sqrt{\frac{1}{O_{11}} + \frac{1}{O_{12}} + \frac{1}{O_{21}} + \frac{1}{O_{22}}} \\ &= \sqrt{0.02898} \\ &= 0.170 \end{aligned}$$

Approximate 95% ci for log odds ratio:

$$\log \text{ OR } \pm (z_{0.975})\,(se)$$

$z_{0.975} = 1.96$

For log odds of cold in Placebo / cold in Vit C:
$\log 1.53 \pm (1.96)(0.17) = 0.427 \pm 0.334 = (0.093,\ 0.761)$
For odds of cold in Placebo / cold in Vit C:
$(\exp(0.093),\ \exp(0.761)) = (1.10,\ 2.14)$
For log odds of cold in Vit C/ cold in Placebo:
$\log 0.652 \pm (1.96)(0.17) = -0.427 \pm 0.334 = (-0.761,\ -0.093,)$
For odds of cold in Placebo / cold in Vit C:
$(\exp(-0.761),\ \exp(-0.093)) = (0.47,\ 0.91)$
Notice that $1/2.14 = 0.47$ and $1/1.10 = 0.91$